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Special Report 190

RESTRAINT PROVIDED A FLAT RECTANGULAR PLATE BY A STURDY STIFFENER ALONG AN EDGE OF THE PLATE

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REPORT NO.______

Comments On

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By

Eugene E. Lundquist
Elbridge Z Stowell

It is believed that the results are seriously limited by the assumption that the cross sections of the stiffener do not deform. For most of our stiffeners, especially bent up sections, the deformations of the cross section are appreciable.

It is shown that, under the assumption of no cross sectional deformation, when the torsional deformation of the stiffener is compatible with the rotation of the panel edge ($\phi = \Phi_0 \cos \pi x/\lambda$), the torsional loading $\frac{dT}{dx}$ on the stiffener at a given station is proportional to the rotation at the given station. Hence, an "equivalent stiffness of the restraint" which is independent of the coordinate x (rotation at one point along length does not affect rotation at another point) can be determined. (See summary.) (WPM)

RESTRAINT PROVIDED A FLAT RECTANGULAR PLATE BY A STURDY STIFFENER ALONG AN EDGE OF THE PLATE

By Eugene E. Lundquist and Elbridge Z. Stowell

SUMMARY

A sturdy stiffener is defined as a stiffener of such proportions that it does not suffer cross-sectional distortion when moments are applied to some part of the cross section. When such a stiffener is attached to one edge of a plate, it will resist rotation of that edge of the plate by means of its torsional properties. A formula is given for the restraint coefficient provided the plate by such a stiffener. This coefficient is required for the calculation of the critical compressive stress of the plate.

INTRODUCTION

In the calculation of the critical compressive stress for flat rectangular plates as presented in references 1 and 2, it is necessary first to evaluate the restraint coefficient along an unloaded edge of the plate. The formula that must be used to evaluate this restraint will depend upon the characteristics of the structural member, or members, which provides the restraint. In references 1 and 2 it was assumed that the restraint was provided by a specially defined elastic restraining medium. As a result of this assumption, it was possible to derive a general design chart for the calculation of the critical compressive stress that is, within wide limits, independent of the structure which provides the restraint.

RESTRAINT COEFFICIENT

In references 1 and 2 the restraint coefficient ϵ is defined, for stresses within the elastic range, by the equation

$$\epsilon = \frac{4S_o b}{D} \qquad (1)$$

and, for stresses beyond the elastic range, by the equation

$$\epsilon = \frac{4S_o b}{\eta D} \tag{2}$$

where

S_o stiffness per unit length of elastic restraining medium at side edge of plate or moment required to rotate a unit length of medium through one-fourth radian

b width of plate being restrained

η nondimensional coefficient to allow for a decrease in D due to the application of stresses beyond the elastic range

D flexural rigidity per unit length of plate $\left[\frac{Et^3}{12(1-\mu^2)}\right]$

E Young's modulus of elasticity

t thickness of plate

μ Poisson's ratio

On the assumption of perfect elasticity, both b and D are constant for the plate and it is necessary to evaluate only S_o. The detailed effect of loading beyond the elastic range is considered in another section of this paper.

The basic property of the elastic restraining medium, as used in the theory of references 1 and 2 in the definition of ϵ, is that rotation at one point of the medium does not affect rotation at another point of the medium. In many practical problems the elastic restraint is provided by a stiffener, a plate, or some other structure for which rotation at one point does affect rotation at another point. Consequently, the evaluation of S_o in any given problem must take into account the effect of this interaction within the elastic restraining structure.

Regardless of how the elastic restraint is provided,

the following two conditions must be satisfied simultaneously:

1. The rotation of the elastic restraining structure at any point along the side edge of the plate must be the same as the rotation of the plate at this point.

2. Each element of the elastic restraining structure must be in equilibrium.

For the solution of the buckling of plates as given in references 1 and 2, the first of these conditions requires that the rotation φ at station x along the side edge of the plate be obtained from equation (A-2) of reference 1 or 2, as follows:

$$\varphi = \varphi_0 \cos \frac{\pi x}{\lambda} \tag{3}$$

where φ_0 is the rotation of the edge at $x = 0$ and λ is the half-wave length of the buckle pattern.

The second of the foregoing conditions requires that the differential equation of equilibrium of the elastic restraining structure be satisfied. If the value of φ that is given by equation (3) also satisfies the equation of equilibrium of the elastic restraining structure, it becomes possible to evaluate the equivalent stiffness S_0 for the elastic restraining structure. A trial calculation showed that this evaluation could be accomplished for the case of a sturdy stiffener.

EVALUATION OF RESTRAINT COEFFICIENT PROVIDED BY A STURDY STIFFENER

A "sturdy" stiffener is defined as a stiffener of such proportions that it does not suffer cross-sectional distortion when the plate buckles. (See fig. 1(b).) Such a stiffener will resist rotation at the side edge of the plate by means of its torsional properties.

Within the elastic range.- According to the definition of S_0, the rotation φ of an element dx of the stiffener ~~mut~~ be caused by an applied moment of $4S_0\varphi dx$. Because a sturdy stiffener can resist this applied moment only by

means of its torsional properties, a difference of torque dT between the ends of the stiffener element dx must exist for equilibrium. Thus, the following differential equation of the equilibrium of any stiffener element dx is obtained by setting the sum of the end moments equal to the applied moment (fig. 2):

$$T - (T + dT) = 4S_o \varphi \, dx \tag{4}$$

from which

$$\frac{dT}{dx} + 4S_o \varphi = 0 \tag{5}$$

A study of the theory of references 3, 4, and 5 reveals that the internal resisting torque T at station x is, for small rotations,

$$T = (GJ - fI_p) \frac{d\varphi}{dx} - EC_{BT} \frac{d^3\varphi}{dx^3} \tag{6}$$

where

GJ torsional rigidity of stiffener

f uniformly distributed compressive stress in stiffener

I_p polar moment of inertia of stiffener sectional area about axis of rotation

C_{BT} torsion-bending constant of stiffener sectional area about axis of rotation at or near edge of plate

(The exact location of the axis of rotation will depend upon the stiffener cross section. The practical evaluation of C_{BT} is reserved for a future paper. In the meantime the reader may consult references 3, 4, 5, and 6 for information concerning C_{BT}.)

Equation (6) of this paper is the same as equation (23) of reference 5, with the addition of another term to include the effect of a uniformly distributed compressive stress f in the stiffener. This stress is also the compressive stress in the sheet because both the stiffener and the sheet are subject to the same deformation.

Substitution of the value of T as given by equation (6) into equation (5) gives

$$EC_{BT}\frac{d^4\varphi}{dx^4} - \left(GJ - fI_p\right)\frac{d^2\varphi}{dx^2} = 4S_o\,\varphi \tag{7}$$

Substitution of equation (3) in equation (7) gives

$$4S_o = \frac{\pi^2}{\lambda^2}\left(GJ - fI_p + \frac{\pi^2}{\lambda^2}EC_{BT}\right) \tag{8}$$

from which

$$\epsilon = \frac{4S_o b}{D} = \frac{\pi^2 b}{\lambda^2 D}\left(GJ - fI_p + \frac{\pi^2}{\lambda^2}EC_{BT}\right) \tag{9}$$

Beyond the elastic range.- When the compressive stress on the plate-and-stiffener combination is beyond the elastic range, equation (9) becomes

$$\epsilon = \frac{\pi^2 b}{\lambda^2 \eta D}\left(\tau_2\,GJ - fI_p + \frac{\pi^2}{\lambda^2}\tau_1\,EC_{BT}\right) \tag{10}$$

where η, τ_2, and τ_1 are nondimensional coefficients less than unity that take into account the effect of stress on D, G, and E, respectively. The problem is to evaluate η, τ_2, and τ_1 for any value of the compressive stress f.

The elemental volumes of the material that comprise the skin-stiffener combination resist buckling by means of the following properties:

1. Longitudinal bending stiffness
2. Torsional stiffness
3. Transverse bending stiffness

At stresses beyond the elastic range, these restraining stiffnesses are multiplied by the nondimensional coefficients τ_1, τ_2, and τ_3, respectively. These coefficients are equal to or less than unity and are given by the following tentative expressions from reference 7:

$$\tau_1 = \tau$$

$$\tau_2 = \frac{\tau + \sqrt{\tau}}{2}$$

$$\tau_3 = 1$$

where τ is $\bar{E}/E$, the ratio of the effective column modulus $\bar{E}$ for longitudinal bending at the stress f to Young's modulus E.

As discussed in reference 7, the coefficient η involves a combination of τ_1, τ_2, and τ_3. A tentative value of η is, in terms of τ,

$$\eta = \frac{\tau + 3\sqrt{\tau}}{4}$$

In the special case, where the plate is an outstanding flange with ϵ very near zero and λ/b is greater than approximately 2.5, the tentative value of η is probably better given by

$$\eta = \tau_2 = \frac{\tau + \sqrt{\tau}}{2}$$

The ratio τ is related to the compressive stress-strain curve for the material. (See equation (11) of reference 7.) In the absence of the compressive stress-strain curve, probable values of τ can be obtained from the column curve for the material. (See reference 7.) Figures 3 and 4 give values of τ_1, τ_2, and η for 24S-T aluminum alloy plotted as a function of the compressive stress f as determined by the column curve for the material. In figure 3, the values apply to 24S-T material with minimum required properties. In figure 4, the values apply to 24S-T material with average properties.

Figures similar to 3 and 4 of this paper may be prepared for any material. The engineer using this paper must therefore decide whether the computation shall be based on the minimum required material properties or the average material properties.

DISCUSSION

In several previous theoretical solutions for the buckling of plates elastically restrained at the side edges, it has been assumed that the restraint is provided only by the torsional rigidity of the stiffener. These solutions neglect two effects: (1) the effect of the increased torsional stiffness caused by longitudinal bending that accompanies torsion and (2), the effect of the reduced torsional stiffness caused by compressive load in the stiffener.

In the solution herein given, these effects have been included and they account for the terms fI_p and $\frac{\pi^2}{\lambda^2} EC_{BT}$ in equation (9). These terms are omitted in equation (83) of reference 8 and in equation (0) on page 343 of reference 9, where rb corresponds to ϵ.

In appendix A of reference 10, the author appears to have realized the importance of including the terms fI_p and $\frac{\pi^2}{\lambda^2} EC_{BT}$. These terms were omitted, however, in the theory that led to the determination of μ of reference 10, which is related to ϵ of this report by the equation

$$\epsilon = \left(\frac{\pi b}{\lambda}\right)^2 \frac{1}{\mu} \tag{11}$$

when fI_p and $\frac{\pi^2}{\lambda^2} EC_{BT}$ are equal to zero.

Theoretical calculations and the results of tests reveal that large errors can result from the omission of some of the terms in equation (9) of this paper. It is therefore recommended that the critical compressive stress for plates elastically restrained against rotation by a sturdy stiffener along one or both unloaded side edges be calculated by the methods of reference 1 or 2 and by equation (9) of this report. It will be found, for channel-, Z-, and I-section stiffeners, that the term in equation (9) which contains C_{BT} will be important; whereas for angle sections this term will be relatively unimportant.

Equations (8) and (9) can also be written:

$$4S_o = \frac{\pi^2 I_p}{\lambda^2}\left[(f_{cr})_{stiff} - f\right] \tag{12}$$

$$\epsilon = \frac{\pi^2 b I_p}{\lambda^2 D}\left[(f_{cr})_{stiff} - f\right] \tag{13}$$

where $(f_{cr})_{stiff}$ is the critical compressive stress for twisting of the stiffener alone. From equation (1) of reference 6,

$$(f_{cr})_{stiff} = \frac{GJ}{I_p} + \frac{\pi^2 E C_{BT}}{\lambda^2 I_p} \tag{14}$$

Equation (13) shows that, when the compressive stress in the stiffener is equal to the critical compressive stress for twisting of the stiffener alone, the restraint coefficient ϵ is equal to zero. When $f < (f_{cr})_{stiff}$, the restraint coefficient ϵ is positive and the stiffener increases the stability of the plate. When $f > (f_{cr})_{stiff}$, the restraint coefficient ϵ is negative and the stiffener decreases the stability of the plate. These results can be inferred because the stiffener cannot aid in stabilizing the plate when it is itself unstable.

Care should be exercised in the application of the formulas of this paper because they apply only to sturdy stiffeners. In practice, no stiffener is ever so sturdy as to be completely without cross-sectional distortion when moments are applied to some part of the cross section.

At present, studies are in progress to evaluate the restraint coefficient provided when the effects of cross-sectional distortion are included. In order to emphasize that all stiffeners suffer some cross-sectional distortion, they are referred to here as "frail" stiffeners in contrast with the hypothetical sturdy stiffeners. (See fig. 1(c).)

As the thickness of the parts in a frail stiffener grows, and all other factors remain unchanged, the action of such a stiffener will approach as a limit the action of a sturdy stiffener. Test data and theory both indicate that whether a given stiffener may be regarded as sturdy or frail depends upon the geometric and the material properties of the attached plate as well as on these properties of the stiffener itself. For example, it was found that in the tests reported in reference 11, the Z-section

stiffener could be safely regarded as sturdy when attached to the sheet that is 0.025 inch thick but must be regarded as frail when attached to the sheet 0.070 inch thick.

CONCLUSIONS

The restraint coefficient ϵ provided a flat rectangular plate by a sturdy stiffener along an edge of the plate is, within the elastic range,

$$\epsilon = \frac{\pi^2 b}{\lambda^2 D}\left(GJ - fI_p + \frac{\pi^2}{\lambda^2} E\, C_{BT}\right)$$

where

b width of plate being restrained

λ half-wave length of buckles

D flexural rigidity per unit length of plate $\left[\frac{Et^3}{12(1 - \mu)^2}\right]$

E Young's modulus of elasticity

t thickness of plate

μ Poisson's ratio

GJ torsional rigidity of stiffener

f uniformly distributed compressive stress in stiffener

I_p polar moment of inertia of stiffener sectional area about axis of rotation

C_{BT} torsion-bending constant of stiffener sectional area about axis of rotation at or near edge of plate

When the plate-stiffener combination is stressed beyond the elastic range, the restraint coefficient is

$$\epsilon = \frac{\pi^2 b}{\lambda^2 \eta D} \left(\tau_2 \; GJ - f I_p + \frac{\pi^2}{\lambda^2} \tau_1 \; E \; C_{BT} \right)$$

where

$\eta = \frac{\tau + 3\sqrt{\tau}}{4}$ approximately

τ ratio of effective modulus $\bar{E}$ to Young's modulus E

$\tau_2 = \frac{\tau + \sqrt{\tau}}{2}$ approximately

$\tau_1 = \tau$

In the special case where the plate is an outstanding flange with ϵ very near zero, and λ/b is greater than approximately 2.5, $\eta = \tau_2 = \frac{\tau + \sqrt{\tau}}{2}$ approximately.

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REFERENCES

1. Lundquist, Eugene E., and Stowell, Elbridge Z.: Critical Compressive Stress for Flat Rectangular Plates Supported along All Edges and Elastically Restrained against Rotation along the Unloaded Edges. NACA confidential report, 1941 (to be published).

2. Lundquist, Eugene E., and Stowell, Elbridge Z.: Critical Compressive Stress for Outstanding Flanges. NACA confidential report, 1941.

3. Wagner, Herbert: Torsion and Buckling of Open Sections. T.M. No. 807, NACA, 1936.

4. Wagner, H., and Pretschner, W.: Torsion and Buckling of Open Sections. T.M. No. 784, NACA, 1936.

5. Kappus, Robert: Twisting Failure of Centrally Loaded Open-Section Columns in the Elastic Range. T.M. No. 851, NACA, 1938.

6. Lundquist, Eugene E., and Fligg, Claude M.: A Theory for Primary Failure of Straight Centrally Loaded Columns. Rep. No. 582, NACA, 1937.

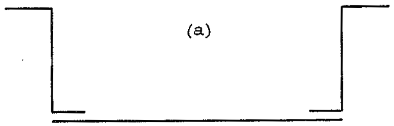
7. Lundquist, Eugene E.: Local Instability of Symmetrical Rectangular Tubes under Axial Compression. T.N. No. 686, NACA, 1939.

8. Trayer, George W., and March, H. W.: Elastic Instability of Members Having Sections Common in Aircraft Construction. Rep. No. 382, NACA, 1931.

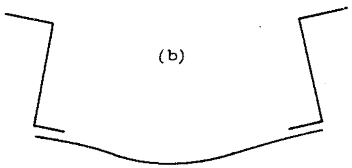
9. Timoshenko, S.: Theory of Elastic Stability. McGraw-Hill Book Co., Inc., New York, 1936.

10. Dunn, Louis G.: An Investigation of Sheet-Stiffener Panels Subjected to Compression Loads with Particular Reference to Torsionally Weak Stiffeners. T.N. No. 752, NACA, 1940.

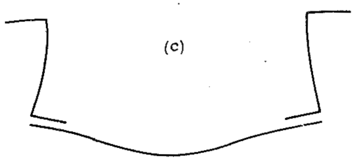
11. Ramberg, Walter, McPherson, Albert E., and Levy, Sam: Experimental Study of Deformation and of Effective Width in Axially Loaded Sheet-Stringer Panels. T.N. No. 684, NACA, 1939.



Before buckling.



After buckling of plate when stiffeners are sturdy (no cross-sectional distortion of stiffeners).



After buckling of plate when stiffeners are frail (cross-sectional distortion of stiffeners).

Figure 1.- Action of sturdy and frail stiffeners when moments are applied to the stiffeners by the buckled plate.

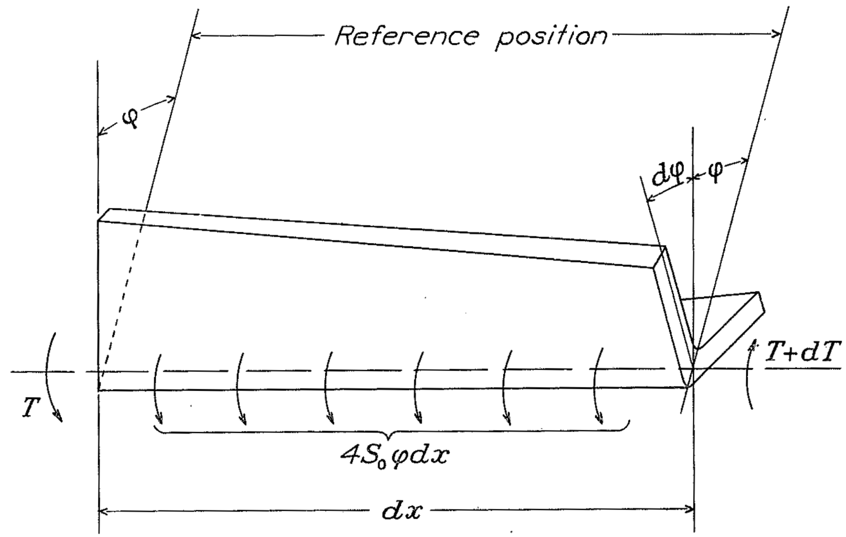


Figure 2.- Equilibrium of a sturdy-stiffener element dx.

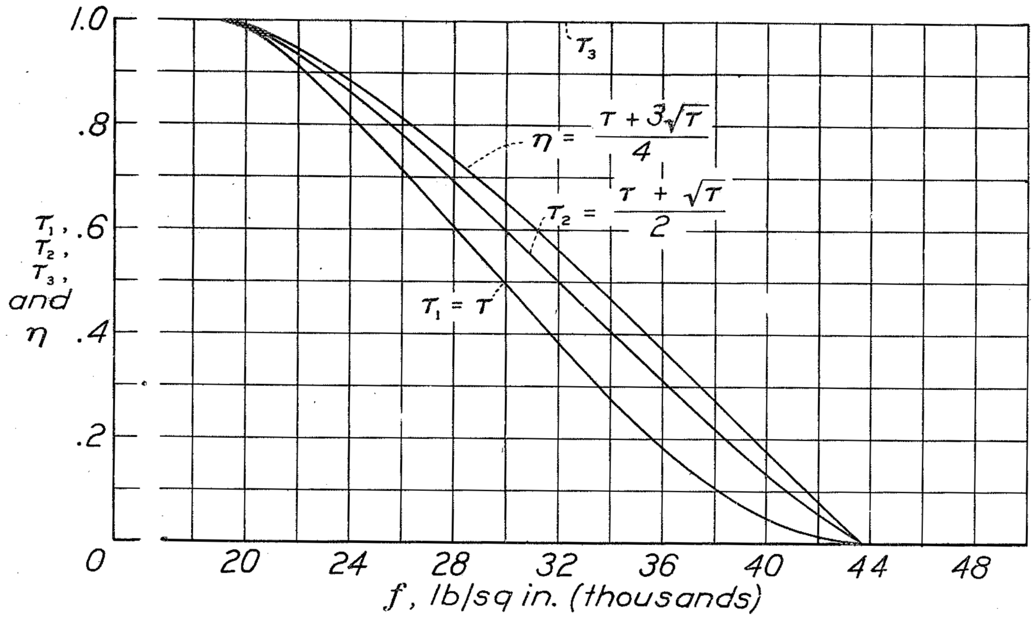


Figure 3.- Variation of τ_1, τ_2, τ_3, and η with the compressive stress, f, for 24S-T aluminum alloy of minimum required properties.

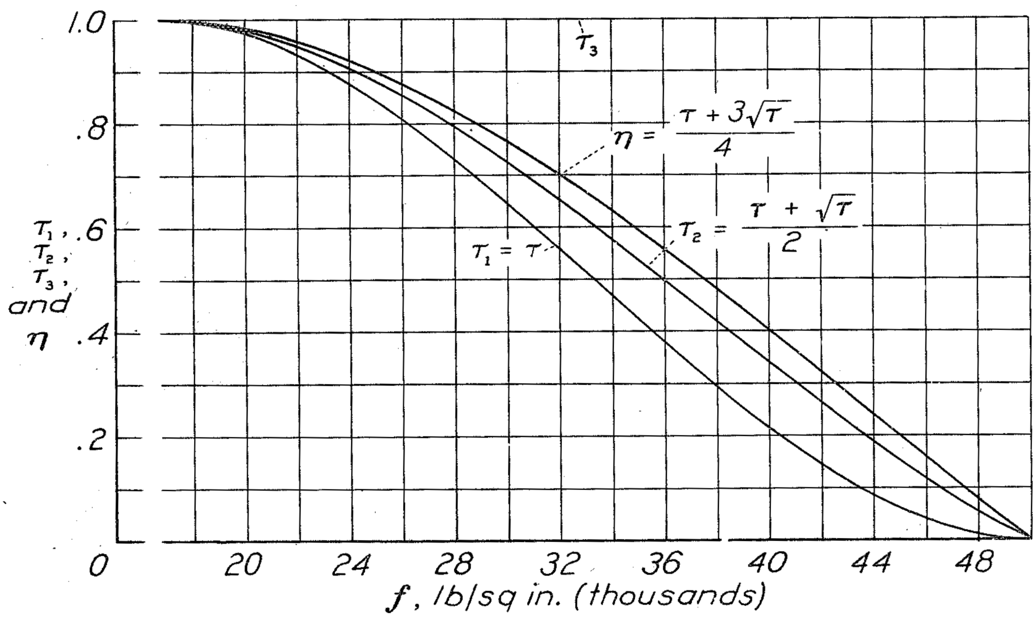


Figure 4.- Variation of τ_1, τ_2, τ_3, and η with the compressive stress, f, for 24S-T aluminum alloy of average properties.